Lecture 4: Complex Numbers Functions, and Data Input

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EE201: Computer Applications. See Textbook Chapter 3.

What is a Function?

- A MATLAB Function (e.g. `y = func(x1, x2)`) is like a script file, but with inputs and outputs provided automatically in the commend window.
- In MATLAB, functions can take zero, one, two or more inputs, and can provide zero, one, two or more outputs.
- There are built-in functions (written by the MATLAB team) and functions that you can define (written by you and stored in .m file).
- Functions can be called from command line, from wihtin a script, or from another function.

Table 3.1–1 Some common mathematical functions

Exponential	
`exp(x)`	Exponential; e^x.
`sqrt(x)`	Square root; $\sqrt{x}$.
Logarithmic	
`log(x)`	Natural logarithm; ln x.
`log10(x)`	Common (base-10) logarithm; log $x = \log_{10} x$.
Complex	
`abs(x)`	Absolute value; x.
`angle(x)`	Angle of a complex number x.
`conj(x)`	Complex conjugate.
`imag(x)`	Imaginary part of a complex number x.
`real(x)`	Real part of a complex number x.
Numeric	
`ceil(x)`	Round to the nearest integer toward ∞.
`fix(x)`	Round to the nearest integer toward zero.
`floor(x)`	Round to the nearest integer toward $-\infty$.
`round(x)`	Round toward the nearest integer.
`sign(x)`	Signum function: $+1$ if $x > 0$; 0 if $x = 0$; -1 if $x < 0$.

Functions are Helpful

- Enable "divide and conquer" strategy
  - Programming task broken into smaller tasks
- Code reuse
  - Same function useful for many problems
- Easier to debug
  - Check right outputs returned for all possible inputs
- Hide implementation
  - Only interaction via inputs/outputs, how it is done (implementation) hidden inside the function.

Finding Useful Functions

- You can use the `lookfor` command to find MATLAB functions that are relevant to your application.
- Example: `>> lookfor imaginary`
- Gets a list of functions that deal with imaginary numbers.
- `i       - Imaginary unit.`
- `j       - Imaginary unit.`
- `complex - Construct complex result from real and imaginary parts.`
- `imag    - Complex imaginary part.`

Calling Functions

- Function names are case sensitive (meshgrid, meshGrid and MESHGRID are interpreted as different functions).
- Inputs (called function *arguments* or function *parameters*) can be either numbers or variables.
- Inputs are passed into the function inside of parentheses `()` separated by commas.
- We usually assign the output to variable(s) so we can use it later. Otherwise it is assigned to the built-in variable `ans`.

Rules

- To evaluate sin 2 in MATLAB, we type `sin(2)`, not `sin[2]`
- For example `sin[x(2)]` gives an error even if x is defined as an array.
- Inputs to functions in MATLAB can be sometimes arrays.

```
>> x = -3 + 4i;
>> mag_x = abs(x)
mag_x =
     5

>> mag_y = abs(6 - 8i)
mag_y =
    10

>> angle_x = angle(x)
angle_x =
    2.2143

>> angle(x)
ans =
    2.2143

>> x = [5,7,15]
x =
     5     7    15

>> y = sqrt(x)
y =
    2.2361    2.6458    3.8730
```


Function Composition

- Composition: Using a function as an argument of another function
- Allowed in MATLAB.
- Just check the number and placement of parentheses when typing such expressions.
- `sin(sqrt(x)+1)`
- `log(x.^2 + sin(5))`

Which expression is correct?

- You want to find $\sin^2(x)$. What do you write?
- `(sin(x))^2`
- `sin^2(x)`
- `sin^2x`
- `sin(x^2)`
- `sin(x)^2`
- *Solution*: Only first and last expressions are correct.

Trigonometric Functions

Trigonometric*	
`cos(x)`	Cosine; cos x.
`cot(x)`	Cotangent; cot x.
`csc(x)`	Cosecant; csc x.
`sec(x)`	Secant; sec x.
`sin(x)`	Sine; sin x.
`tan(x)`	Tangent; tan x.
Inverse trigonometric†	
`acos(x)`	Inverse cosine; arccos $x = \cos^{-1} x$.
`acot(x)`	Inverse cotangent; arccot $x = \cot^{-1} x$.
`acsc(x)`	Inverse cosecant; arccsc $x = \csc^{-1} x$.
`asec(x)`	Inverse secant; arcsec $x = \sec^{-1} x$.
`asin(x)`	Inverse sine; arcsin $x = \sin^{-1} x$.
`atan(x)`	Inverse tangent; arctan $x = \tan^{-1} x$.
`atan2(y,x)`	Four-quadrant inverse tangent.

*These functions accept x in radians.
†These functions return a value in radians.

Hyperbolic functions

Hyperbolic	
`cosh(x)`	Hyperbolic cosine; cosh $x = (e^x + e^{-x})/2$.
`coth(x)`	Hyperbolic cotangent; cosh x/sinh x.
`csch(x)`	Hyperbolic cosecant; 1/sinh x.
`sech(x)`	Hyperbolic secant; 1/cosh x.
`sinh(x)`	Hyperbolic sine; sinh $x = (e^x - e^{-x})/2$.
`tanh(x)`	Hyperbolic tangent; sinh x/cosh x.
Inverse hyperbolic	
`acosh(x)`	Inverse hyperbolic cosine
`acoth(x)`	Inverse hyperbolic cotangent
`acsch(x)`	Inverse hyperbolic cosecant
`asech(x)`	Inverse hyperbolic secant
`asinh(x)`	Inverse hyperbolic sine
`atanh(x)`	Inverse hyperbolic tangent

User-Defined Functions

- Functions must be saved to a file with .m extension.
- Filename (without the .m) must match EXACTLY the function name.
- First line in the file must begin with a function definition line that illustrates inputs and outputs.

```
function [output variables] = name(input variables)
```

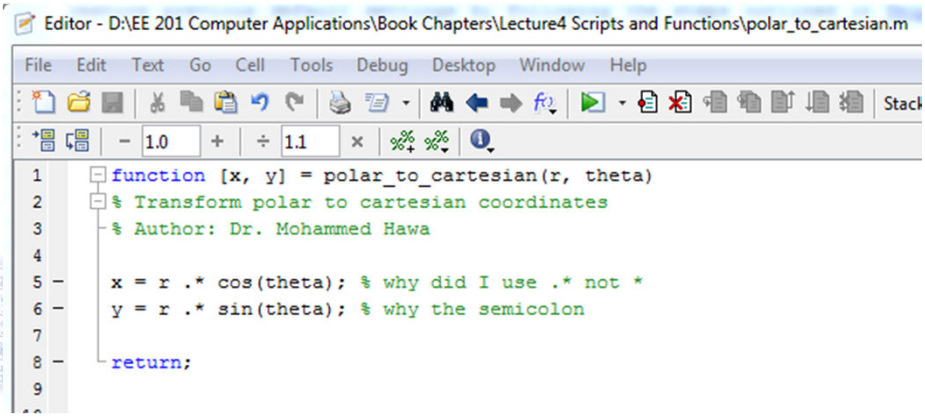
- This line distinguishes a function M-file from a script M-file.
- Output variables are enclosed in square brackets.
- Input variables must be enclosed with parentheses.

Functions Names

- Function names may only use alphanumeric characters and the underscore.
- Functions names should NOT:
  - include spaces
  - start with a number
  - use the same name as an existing command
- Consider adding a header comment, just under the function definition (for `help`).

Exercise: Your Own `pol2cart`

- Make sure you set you Current Folder to Desktop (or where you saved the .m file).

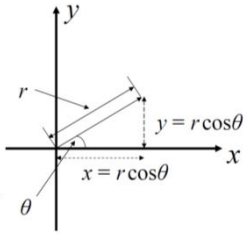


Test your newly defined function

```
>> [a, b] = polar_to_cartesian(3, pi)
a =
    -3
b =
  3.6739e-016

>> polar_to_cartesian(3, pi)
ans =
    -3

>> [a, b] = polar_to_cartesian(3, pi/4)
a =
    2.1213
b =
    2.1213

>> [a, b] = polar_to_cartesian([3 3 3], [pi pi/4 pi/2])
a =
   -3.0000    2.1213    0.0000
b =
    0.0000    2.1213    3.0000
```


MATLAB has `pol2cart`

```
>> help pol2cart

 POL2CART Transform polar to Cartesian coordinates.
    [X,Y] = POL2CART(TH,R) transforms corresponding elements of data
    stored in polar coordinates (angle TH, radius R) to Cartesian
    coordinates X,Y.  The arrays TH and R must the same size (or
    either can be scalar).  TH must be in radians.

    [X,Y,Z] = POL2CART(TH,R,Z) transforms corresponding elements of
    data stored in cylindrical coordinates (angle TH, radius R, height
    Z) to Cartesian coordinates X,Y,Z. The arrays TH, R, and Z must be
    the same size (or any of them can be scalar).  TH must be in radians.

    Class support for inputs TH,R,Z:
       float: double, single

    See also cart2sph, cart2pol, sph2cart.

    Reference page in Help browser
       doc pol2cart
```

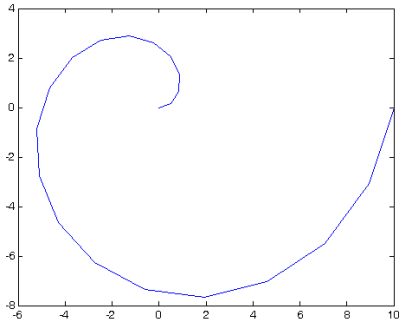


Just like your code!

```
>> type pol2cart

function [x,y,z] = pol2cart(th,r,z)
%POL2CART Transform polar to Cartesian coordinates.
%   [X,Y] = POL2CART(TH,R) transforms corresponding elements of data
%   stored in polar coordinates (angle TH, radius R) to Cartesian
%   coordinates X,Y.  The arrays TH and R must the same size (or
%   either can be scalar).  TH must be in radians.
%
%   [X,Y,Z] = POL2CART(TH,R,Z) transforms corresponding elements of
%   data stored in cylindrical coordinates (angle TH, radius R, height
%   Z) to Cartesian coordinates X,Y,Z. The arrays TH, R, and Z must be
%   the same size (or any of them can be scalar).  TH must be in radians.
%
%   Class support for inputs TH,R,Z:
%      float: double, single
%
%   See also CART2SPH, CART2POL, SPH2CART.

%   L. Shure, 4-20-92.
%   Copyright 1984-2004 The MathWorks, Inc.
%   $Revision: 5.9.4.2 $  $Date: 2004/07/05 17:02:08 $

x = r.*cos(th);
y = r.*sin(th);
```

Exercise: Spiral

```
>> r = linspace(0, 10, 20);
>> theta = linspace(0, 2*pi, 20);
>> [x, y] = polar_to_cartesian(r, theta);
>> plot(x,y);
```



Possible Cases

- One input:

```
function [o1, o2, o3] = myfunc(i1)
```

- Three inputs:

```
function [o1, o2, o3] = myfunc(i1, i2, i3)
```

- No inputs:

```
function [o1, o2, o3] = myfunc()
function [o1, o2, o3] = myfunc
```

- One output:

```
function [o1] = myfunc(i1, i2, i3)
function o1 = myfunc(i1, i2, i3)
```

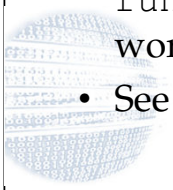
- No output:

```
function [] = myfunc(i1, i2, i3)
function myfunc(i1, i2, i3)
```


Local Variables

```
function z = fun(x,y)

u = 3*x;
z = u + 6*y.^2;

% return missing is fine at end of file
```

- The variables x, y, u, z are **local** to the function `fun`, so their values will not be available in the workspace outside the function.
- See example below.



Example

```
>> x = 3;
>> b = 7;
>> q = fun(x, b);

>> x
x =
     3

>> y
??? Undefined function or variable 'y'.

>> u
??? Undefined function or variable 'u'.

>> z
??? Undefined function or variable 'z'.

>> q
q =
   303
```

Exercise

```
function show_date
clear
clc
date

% how many inputs and outputs do we have?
```


Homework

```
function [dist, vel] = drop(v0, t)
% Compute the distance travelled and the
% velocity of a dropped object, from
% the initial velocity v0, and time t
% Author: Dr. Mohammed Hawa

g = 9.80665; % gravitational acceleration (m/s^2)
vel = g*t + v0;
dist = 0.5*g*t.^2 + v0*t;
```

```
>> t = 0:0.1:5;
>> [distance_dropped, velocity] = drop(10, t);
>> plot(t, velocity)
```


Local vs. Global Variables

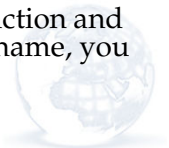
- The variables inside a function are local. Their scope is only inside the function that declares them.
- In other words, functions create their own workspaces.
- Function inputs are also created in this workspace when the function starts.
- Functions do not know about any variables in any other workspace.
- Function outputs are copied from the function workspace when the function ends.
- Function workspaces are destroyed after the function ends.
  - Any variables created inside the function "disappear" when the function ends.

Local vs. Global Variables

- You can, however, define global variables if you want using the `global` keyword.
- Syntax: `global a x q`
- Global variables are available to the basic workspace and to other functions that declare those variables global (allowing assignment to those variables from multiple functions).

Subfunctions

- An M-file may contain more than one user-defined function.
- The first defined function in the file is called the *primary* function, whose name is the same as the M-file name.
- All other functions in the file are called *subfunctions*. They can serve as subroutines to the primary function.
- Subfunctions are normally "visible" only to the primary function and other subfunctions in the same file; that is, they normally cannot be called by programs or functions outside the file.
- However, this limitation can be removed with the use of function handles.
- We normally use the same name for the primary function and its file, but if the function name differs from the file name, you must use the file name to invoke the function.



Exercise

- The following example shows how the MATLAB M-function `mean` can be superceded by our own definition of the mean, one which gives the root-mean square value.

```
function y = subfun_demo(a)
y = a - mean(a);

function w = mean(x)
w = sqrt(sum(x.^2))/length(x);
```


Example

- A sample session follows.

```
>>y = subfn_demo([4 -4])
y =
     1.1716     -6.8284
```

- If we had used the MATLAB M-function mean, we would have obtained a different answer; that is,

```
>>a = [4 -4];
>>b = a - mean(a)
b =
     4      -4
```


Function Handles

- You can create a function handle to any function by using the @ sign before the function name.
- You can then use the handle to reference the function.

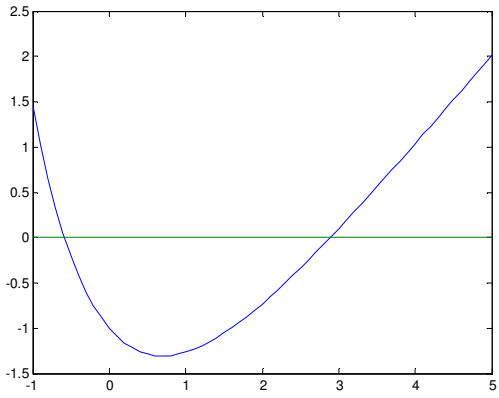
```
function y = f1(x)
y = x + 2*exp(-x) - 3;
```

- You can pass the function as an argument to another function using the handle. Example: `fzero` function finds the zero of a function of a single variable x.
- `>> x0 = 3; % initial guess`
- `>> fzero(@f1, x0)`

Handle vs. Return Value

```
t = -1:0.1:5;
plot(t, f1(t));
```

- There is a zero near $x = -0.5$ and one near $x = 3$.



Exercise

`fzero(@function,x0)`

- where @function is the function handle, and x0 is a user-supplied initial guess for the zero.

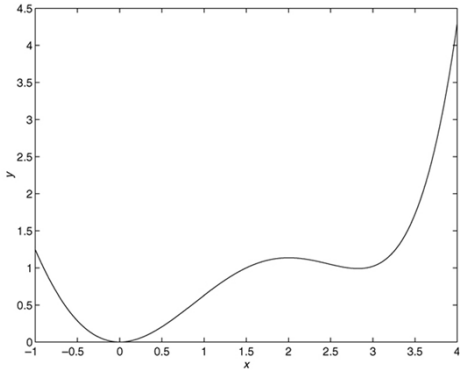
```
>> fzero(@f1, -0.5)
ans =
   -0.5831

>> fzero(@f1, 3)
ans =
    2.8887

>> fzero(@sin, 0.1)
ans =
  6.6014e-017

>> fzero(@cos, 2)
ans =
    1.5708

>> pi/2
ans =
    1.5708
```


Finding the Minimum

- The `fminbnd` function finds the minimum of a function of a single variable x in the interval x1 ≤ x ≤ x2.
- `fminbnd(@function, x1, x2)`
- `fminbnd(@cos, 0, 4)` returns 3.1416
- `function y = f2(x)`
- `y = 1-x.*exp(-x);`
- `x = fminbnd(@f2, 0, 5)` returns x = 1
- How would I find the min value of f2? (i.e. 0.6321)

Exercise

- For the function:
- $y = 0.025x^5 - 0.0625x^4 - 0.333x^3 + x^2$
- Find the minimum in the intervals:
- $x \in [-1, 4]$
- $x \in [1, 4]$
- $x \in [2, 4]$
- $x \in [-1, 1]$



Old vs. New

- New syntax for function handles:

```
fzero(@f1, -0.5)
```

- Older syntax for function handles :

```
fzero('f1', -0.5)
```

- The new syntax is preferred, though both will work just fine.
- Which one gives the correct answer: `fzero('sin', 3)` or `fzero(@sin, 3)`

The `fminsearch` function

- `fminsearch` finds minimum of a function of *more than one* variable.
- To find where the minimum of $f = xe^{-(x^2+y^2)}$, define it in an M-file, using the vector x whose elements are x(1) = x and x(2) = y.

```
function f = f4(x)
f = x(1).*exp(-x(1).^2-x(2).^2);
```

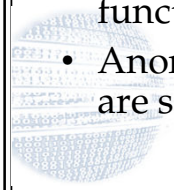
- Suppose we guess that the minimum is near $x = 0, y = 0$.

```
>>fminsearch(@f4,[0,0])
ans =
   -0.7071    0.000
```

- Thus the minimum occurs at $x = -0.7071, y = 0$.

Inline Function

- No need to save the function in an M-file.
- Useful for small size functions defined on the fly.
- You can use a string array to define the function.
- Anonymous functions are similar (see next).



```
>> f4 = inline('x.^2-4')
f4 =
     Inline function:
     f4(x) = x.^2-4

>> [x, value] = fzero(f4, 0)
x =
    -2
value =
     0

>> f5str = 'x.^2-4'; % string array
>> f5 = inline(f5str)
f5 =
     Inline function:
     f5(x) = x.^2-4

>> x = fzero(f5, 3)
x =
     2

>> x = fzero('x.^2-4', 3)
x =
     2

>> f6 = inline('x.*y')
f6 =
     Inline function:
     f6(x,y) = x.*y
```


Anonymous functions

- Here is a simple function called `sq` to calculate the square of a number.

```
>> sq = @(x) x.^2;

>> sq = @(x) (x.^2)
sq =
    @(x)(x.^2)

>> sq([5 7])
ans =
    25    49

>> fminbnd(sq, -10, 10)
ans =
     0
```

Exercise

```
>> sqrtsum = @(x,y) sqrt(x.^2 + y.^2);

>> sqrtsum(3, 4)
ans =
     5

>> A = 6; B = 4;

>> plane = @(x,y) A*x + B*y;

>> z = plane(2,8)
z =
    44

>> f = @(x) x.^3; % try by hand!
>> g = @(x) 5*sin(x);
>> h = @(x) g(f(x));
>> h(2)
ans =
    4.9468
```


Variables in Anonymous Functions

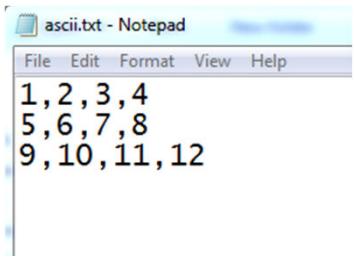
- When the function is created MATLAB, it captures the values of these variables and retains those values for the lifetime of the function handle. If the values of A or B are changed after the handle is created, their values associated with the handle do not change.
- This feature has both advantages and disadvantages, so you must keep it in mind.

For Speed Use Handles

- The function handle provides speed improvements.
- Another advantage of using a function handle is that it provides access to subfunctions, which are normally not visible outside of their defining M-file.

Importing Data: ASCII

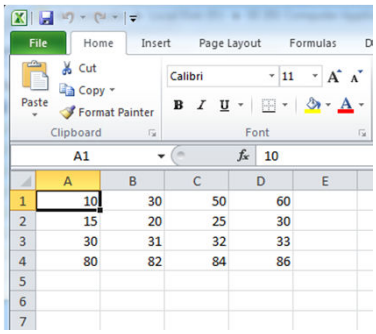
- Make the 'data' folder your Current Folder.
- Delimited ASCII files are common to save data from experiments.
- `dlmread/dlmwrite`




```
>> a = dlmread('ascii.txt')
a =
     1     2     3     4
     5     6     7     8
     9    10    11    12
```


Importing Data: Excel

- Make the 'data' folder your Current Folder.
- MATLAB can also read and write to Excel Files.
- `xlsread/xlswrite`




```
>> a = xlsread('data.xls')
a =
    10    30    50    60
    15    20    25    30
    30    31    32    33
    80    82    84    86
```


Importing Data: Images

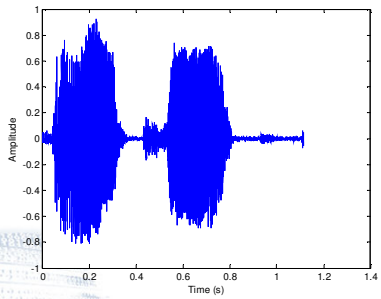
- Make the 'data' folder your Current Folder.
- Read and write images:
- `imread/imwrite`

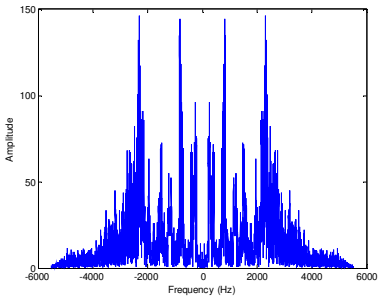
```
>> c = imread('cat.jpg');
>> imshow(c);
>>
>> imshow(255-c); % inverse
```


Importing Data: Sound Files

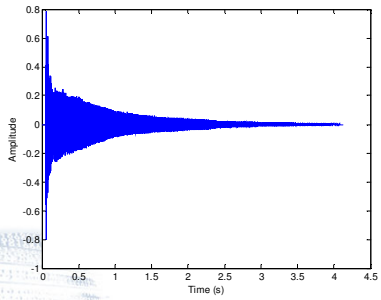
```
% use a script file (fourier.m)
[y,Fs,bits] = wavread('bequiet');
N = length(y);
t = (1/Fs)*(1:N);

plot(t, y);
xlabel('Time (s)');
ylabel('Amplitude');

f = Fs*(-N/2:N/2-1)/N;
y_fft = fftshift(abs(fft(y)));

figure;
plot(f, y_fft);
xlabel('Frequency (Hz)');
ylabel('Amplitude');
```

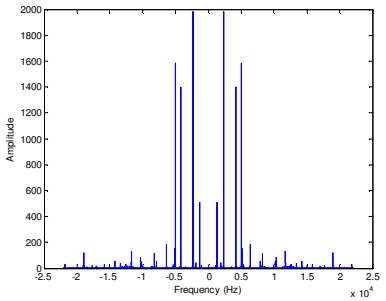

bequiet.wav (BW of human voice!)



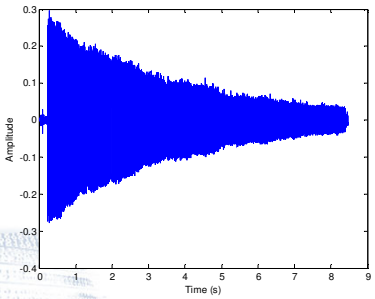


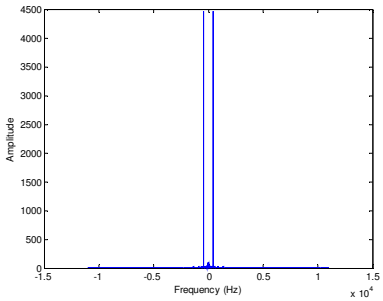
triangle.way

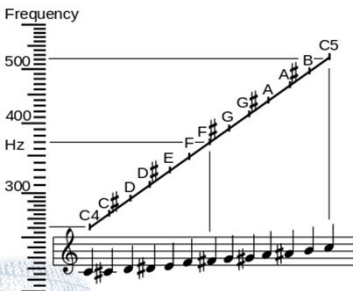




tuningA4.wav (frequency?)







MIDI number	Note name	Keyboard	Frequency Hz		Period ms	
21	A0		27.500		36.36	
22	B0		30.868	29.135	32.40	34.32
23	C1		32.703		30.58	
24	D1		36.708	34.648	27.24	28.86
25	E1		41.203	38.891	24.27	25.71
26	F1		43.654		22.91	
27	G1		48.999	46.249	20.41	21.62
28	A1		55.000	51.913	18.18	19.26
29	B1		61.735	58.270	16.20	17.16
30	C2		65.406		15.29	
31	D2		73.416	69.296	13.62	14.29
32	E2		82.407	77.782	12.13	12.86
33	F2		87.307		11.45	
34	G2		97.999	92.499	10.20	10.81
35	A2		110.00	103.83	9.091	9.631
36	B2		123.47	116.54	8.099	8.581
37	C3		130.81		7.645	
38	D3		146.83	138.59	6.811	7.216
39	E3		164.81	155.56	6.068	6.428
40	F3		174.61		5.727	
41	G3		196.00	185.00	5.102	5.405
42	A3		220.00	207.65	4.545	4.816
43	B3		246.94	233.08	4.050	4.290
44	**C4**		**261.63**		**3.822**	
45	D4		293.67	277.18	3.405	3.608
46	E4		329.63	311.13	3.034	3.214
47	F4		349.23		2.863	
48	G4		392.00	369.99	2.551	2.703
49	**A4**		**440.00**	415.30	**2.273**	2.408
50	B4		493.88	466.16	2.025	2.145
51	C5		523.25		1.910	
52	D5		587.33	554.37	1.703	1.804
53	E5		659.26	622.25	1.517	1.607
54	F5		698.46		1.432	
55	G5		783.99	739.99	1.276	1.351
56	A5		880.00	830.61	1.136	1.204
57	B5		987.77	932.33	1.012	1.073
58	C6		1046.5		0.9556	
59	D6		1174.7	1108.7	0.8513	0.9020
60	E6		1318.5	1244.5	0.7584	0.8034
61	F6		1396.9		0.7159	
62	G6		1568.0	1480.0	0.6378	0.6757
63	A6		1760.0	1661.2	0.5682	0.6020
64	B6		1975.5	1864.7	0.5062	0.5363
65	C7		2093.0		0.4778	
66	D7		2349.3	2217.5	0.4257	0.4510
67	E7		2637.0	2489.0	0.3792	0.4018
68	F7		2793.0		0.3580	
69	G7		3136.0	2960.0	0.3189	0.3378
70	A7		3520.0	3322.4	0.2841	0.3010
71	B7		3951.1	3729.3	0.2531	0.2681
72	C8	J. Wolfe, UNSW	4186.0		0.2389	
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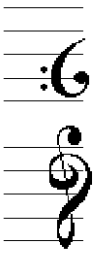
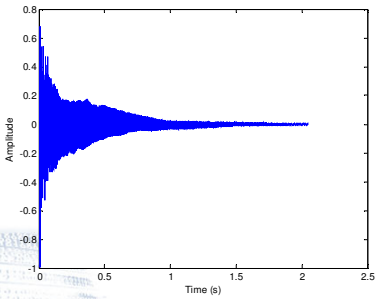
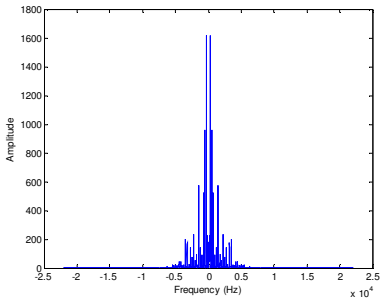


guitar.wav





Homework

- Solve as many problems from Chapter 3 as you can
- Suggested problems:
- 3.1, 3.3, 3.6, 3.9, 3.14, 3.18, 3.24

